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Disorder and the fractional quantum Hall effect: the reduction of the gap

A Mielke
Institut de Physique Théorique, Ecole Polytechnique Fédérale de Lausanne, PHB-Ecublens, CH-1015 Lausanne, Switzerland

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Abstract. We present numerical as well as analytical results for the reduction of the energy gap in the fractional quantum Hall regime due to disorder. The disordered substrate is treated perturbatively. It consists of randomly distributed long-range scatterers. It turns out that the reduction of the energy gap is determined by the mean value of the fluctuations of the potential alone and that it is proportional to the fractional charge of the quasiparticles.

1. Introduction

The fractional quantum Hall effect (for a recent review see [1]) is usually explained by the fact that at certain densities, the ground state of the two-dimensional electron gas is exceptionally stable. The idea of Laughlin [2] was to introduce fractionally charged quasiparticles or quasiholes. Slight deviations from a stable density would create those quasiparticles or quasiholes. Between the ground state at the stable density and the states with quasiparticles or quasiholes, there is a finite energy gap, the ground state is incompressible. The energy gap is related to a discontinuity in the chemical potential. It was determined experimentally from the thermal activation of the diagonal resistivity [3, 4] and from magneto-optical measurements [5]. Theoretically, the energy gap was calculated using various methods (see e.g. [1]). The main problem is that the theoretical calculations lead to values of the energy gap which are much larger than the experimentally determined values. There are several mechanisms for the reduction of the energy gap. The main effect is due to the finite spread of the electron wavefunction perpendicular to the two-dimensional plane (chapter 2.6 in [1]). Further there is a reduction of the energy gap due to Landau level mixing [6] and due to disorder. Recent experiments on systems with very high mobilities [4] show that the effect of disorder is not negligible. Theoretically the effect of disorder on the energy gap has been discussed using different methods. The effect of a single impurity was numerically calculated by Zhang *et al* [7] and by Rezayi *et al* [8]. MacDonald *et al* [9] and Gold [10] gave analytical estimates of the reduction of the energy gap due to disorder using a single mode approximation. Their results contain parameters that are used to fit the experimental data.

In the following we will calculate the reduction of the energy gap using a perturbative approach. Our article is organized as follows. In the following section we give expressions for the energy gap and its relation to the chemical potential. In section 3 we introduce

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two different kinds of substrate potentials that are used in the numerical calculations. In section 4 we present the numerical results for the reduction of the energy gap due to disorder. The disordered substrate consists of long-range scatterers, e.g. Coulomb scatterers or Gaussian scatterers. It turns out that the reduction of the energy gap depends for slowly varying potentials of this kind only on the mean value of the fluctuations of the potential. In section 5 we will present some analytical results to explain this behaviour. We will show that the reduction of the energy gap is proportional to the fractional charge of the quasiparticles. In section 6 we will compare our results with the experimental and theoretical results mentioned above and in section 7 we present conclusions. Throughout we will use units where the cyclotron radius is set to unity.

2. The energy gap

In the following we will discuss a model of two-dimensional interacting electrons on a torus in a strong magnetic field. For details of the model we refer to [11, 12]. The energy gap is usually calculated as the sum of the excitation energies of the quasiparticle and the quasihole [13]. The excitation energies may be written as

$$E_{\pm} = E_0(N_s \pm 1, N_e) - E_0(N_s, N_e) \tag{1}$$

where $E_0(N_s, N_e)$ is the ground state energy of a system with N_s flux quanta and N_e electrons. One obtains the energy gap at a filling factor $f_0 = N_e/N_s = q/p$ (q, p relative prime, p odd and small) from (1)

$$\delta E(f_0) = E_+ + E_-. \tag{2}$$

The energy gap is related to a discontinuity in the chemical potential [13] which is introduced using

$$\mu_+ = E_0(N_s, N_e + 1) - E_0(N_s, N_e) \tag{3a}$$

$$\mu_- = E_0(N_s, N_e) - E_0(N_s, N_e - 1) \tag{3b}$$

so that at a filling factor $f_0 = 1/p$, one has

$$\delta\mu(f_0) = \mu_+ - \mu_- = E_0(N_s, N_e + 1) + E_0(N_s, N_e - 1) - 2E_0(N_s, N_e) \approx p\delta E(f_0). \tag{4}$$

The last estimate in (4) follows, since adding one electron (hole) to the system corresponds to adding p quasi-electrons (quasiholes). This is in fact only true if the quasiparticles (quasiholes) are well separated so that the interaction between them might be neglected.

Another way to calculate the energy gap is used for instance in [7–10]. Here the energy gap is calculated for fixed N_e and N_s. This means that neutral excitations are taken into account. They are interpreted as magneto-rotons, magneto-excitons or quasi-excitons. A quasi-exciton is built out of a quasiparticle and a quasihole and its energy is given by (2) as long as the interaction between the quasiparticles is negligible. The energies calculated within these approaches are in good agreement with the energies that are calculated as described in (2). Magneto-optical measurements [5] have some relation to the picture of neutral excitations. On the other hand, transport measurements [4] are more easily related to charged excitations. Both measurements yield more or less the same value for the energy gap. But it is not *a priori* clear that the effect of disorder on different types of excitations is the same.

In a small system without disorder the ground state energies at different filling factors may easily be calculated numerically. Due to the symmetry of the system, the ground state energies are degenerate. By introducing a disordered background potential this degeneracy is removed. In the thermodynamic limit, the ground state energy of a system at a filling factor f_0 remains degenerate. In small systems, the splitting of the ground state energy at such a filling factor is small compared to the splitting of the ground state energy at a filling factor $f_\pm = N_e/(N_s \pm 1)$ [11]. The reduction of the energy gap due to disorder may be calculated using a degenerate perturbation theory and is given by the sum of the reductions of the ground state energies at the filling factors $f_\pm$. To first order, one has to calculate the matrix elements of the disordered substrate between the degenerate ground states and to diagonalize the resulting matrix. Details may be found in [11].

3. Substrate potentials

Walukiewicz *et al* [14] discussed the main scattering mechanisms in modulation-doped heterostructures. In GaAs–GaAlAs heterostructures at low temperature, the main scattering mechanism is the scattering by ionized impurities. The main part of the impurities is situated in the doped region of the heterostructure (remote scattering). The authors claim that scattering mechanisms such as surface roughness (that is important in Si-MOSFETs) or interface charge scattering do not play a significant role. This means that the disordered substrate potential may be written as in [9]

$$V_q = \sum_j \exp(-\mathrm{i}\boldsymbol{q}\boldsymbol{R}_j) U(|\boldsymbol{q}|, d_j, z_0) \tag{5a}$$

$$U(q, d, z) = 2\pi e^2 \exp(-qd)/\varepsilon q(1 + qz/3)^3. \tag{5b}$$

Here V_q is the Fourier transform of the substrate potential. The sum in (5*a*) runs over all impurities. $\boldsymbol{R}_j$ are the coordinates of the jth impurity projected onto the plane of the two-dimensional electron gas (here the xy plane), d_j is the distance of the jth impurity from the plane. The parameter z_0 in (5*a*) describes the z extension of the electron wavefunction in a variational Stern–Howard approach (see e.g. [15]). The coordinates R_j are distributed uniformly and the distances d_j are distributed uniformly between d and $d + a$ where d is the spacer thickness and a is the thickness of the doped region. If one performs averages, one has to include some correlations in the positions of the scatterers to exclude macroscopic fluctuations of the substrate potential. This may be done by introducing a structure factor for the distribution of the R_j [9]. In the numerical calculations described below such correlations are not necessary.

For comparison we have also used background potentials that consist of an equal amount of attractive and repulsive Gaussian scatterers randomly distributed in the xy plane. In this case the Fourier components of the substrate are given by

$$V_q = \sum_j V(-)^j \exp(-d^2q^2 - \mathrm{i}\boldsymbol{q} \cdot \boldsymbol{R}_j). \tag{6}$$

Potentials of this kind are often used in numerical calculations since they are easily calculated and contain only a single parameter d, which describes the range of the

scatterers. To compare the results with the results for Coulomb scatterers, we will use long-range Gaussian scatterers. The parameter d will be chosen to be larger than 1.

4. Numerical results

We have calculated the reduction of the energy gap due to a disordered substrate potential (5) to first order in a perturbational approach. This was done for small systems with periodic boundary conditions. In a first step the multi-particle Hamiltonian with a Coulomb interaction between the electrons but without the disordered substrate potential was diagonalized within a Hilbert space restricted to the lowest Landau level. Due to the symmetry of the system the ground states are degenerate. At filling factors near but not equal to q/p (p odd and small) the degeneracy is of the order of the system size. The splitting of the ground state energy due to the disordered substrate was calculated within the first order of a degenerate perturbation expansion (see e.g. [16]). The excitation energies in (1) are shifted to lower values. This yields a reduction of the energy gap given in (2). Further details may be found in [11].

The substrate potentials consist of Coulomb scatterers as described in section 3. The parameters are $a = 2$; $d = 2, 3$ or 4, $z_0 = 0, 1, 2$ or 3 in units of the cyclotron radius. The density of scatterers n_{sc} takes the values $0.5, 0.75, 1, 1.5, 2$. For each triple (d, z_0, n_{sc}) 20 potentials have been calculated using a usual random number generator for the uniformly distributed coordinates $\boldsymbol{R}_j$ and the distances d_j.

In figure 1 the reduction of the energy gap in a system with a filling factor $1/3$ (five electrons, figure 1(a)) and $1/5$ (four electrons, figure 1(b)) is plotted for 1200 different substrate potentials as a function of Γ which is given by

$$\Gamma^2 = \sum_q |V_q|^2 \exp(-q^2/2). \tag{7}$$

Γ is well known, it is the parameter that describes the Landau level broadening within a self-consistent Born approximation (see e.g. section VI.A. in [15]). In the case of slowly varying potentials, Γ describes the magnitude of the potential fluctuations. In our case it turns out that the reduction of the energy gap is approximately given by

$$\Delta = C_f \Gamma \tag{8}$$

where the constant C_f is roughly 0.7 for a filling factor $f = 1/3$ and 0.5 for $f = 1/5$. This result does not depend on the size of the system. We have calculated the reduction of the energy gap for different system sizes (3, 4, 5 and 6 electrons in the case $f = 1/3$, 3 and 4 electrons in the case $f = 1/5$) and found that the results are essentially the same.

As long as d is not too small, the result is independent of the parameters d, a and z_0 that characterize the disordered substrate potential. If d is smaller than the cyclotron radius, the fluctuations of the reduction of the energy gap around the value (8) become large. This means that the reduction of the energy gap is no longer a function of Γ alone.

For comparison we have calculated the reduction of the energy gap for a potential that is given by (6). The results are presented in figure 2(a) for $f = 1/3$ and in figure 2(b) for $f = 1/5$ for 1200 potentials. They are essentially the same as before, the reduction of the energy gap is given by (8). In this case we have chosen $d = 2.5, 3.5$ or 4.5 and n_{sc} as above. For smaller values of d the fluctuations around the value (8) become too large. For each pair (d, n_{sc}) 80 different random potentials were calculated. In general the fluctuations around the value (8) are larger for $f = 1/5$ than for $f = 1/3$. The reason is

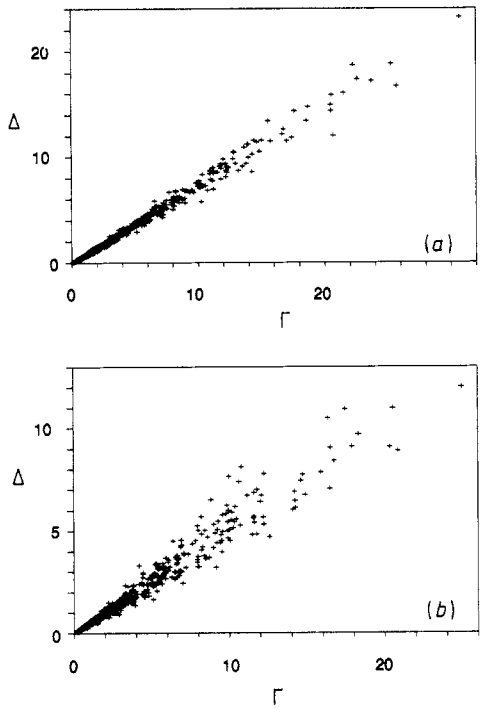

Figure 1. The reduction of the energy gap versus Γ for 1200 different potentials that consist of Coulomb scatterers. The density of the scatterers is varied between 0.5 and 2, d is varied between 2 and 4, z_0 is varied between 0 and 3, and $a = 2$. The filling factor is (a) $f = 1/3$ and (b) $f = 1/5$. Energies are given in units of $e^2/\varepsilon l$, where l is the cyclotron radius.

that the range of the fluctuations of the substrate potential has to be compared with a typical extension of the fractionally charged quasiparticles or quasiholes in space. They are smaller for $f = 1/3$ than for $f = 1/5$.

5. An analytical estimate of the reduction of the energy gap

The reduction of the energy gap may be calculated analytically using (4). The energies $E_0(N_s, N_e \pm 1)$ are highly degenerate in a system with a homogeneous background. The splitting of the degeneracy due to a disordered background may be calculated within a perturbational approach. This was done in [12]. It turns out that the additional electron or hole behaves as a single particle in the lowest Landau level moving in an effective potential. It may be described by an effective single particle Hamiltonian. The effective potentials are given by (equations (3.9) and (3.19) in [12])

$$V_+(\boldsymbol{x}) = \sum_k V_k(N_s + t_k)/(N_s - N_e) \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}) \tag{9}$$

for the additional electron and

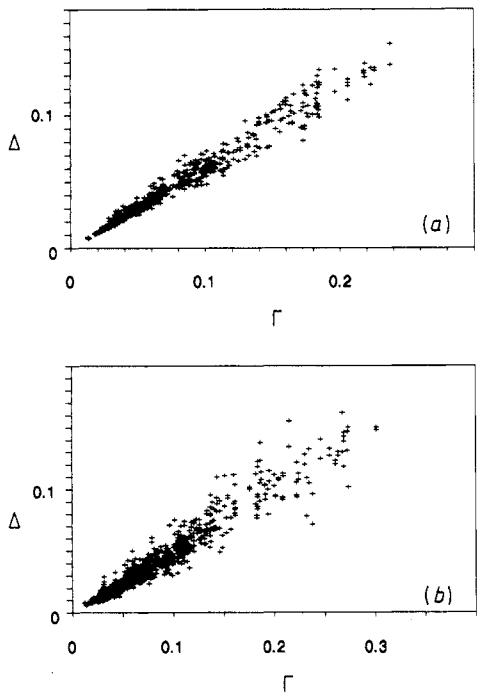

Figure 2. The same as in figure 1 for 1200 different potentials that consist of Gaussian scatterers with an amplitude V. d is varied between 2.5 and 4.5. The energies are given in units of V.

$$V_-(\boldsymbol{x}) = \sum_k V_k(t_k/N_e)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}) \tag{10}$$

for the hole. V_k are the Fourier components of the substrate potential and t_k are related to the static structure factor s_k of the ground state at the filling factor f_0

$$t_k = N_e(s_k - 1 + N_e\delta_{k,0}]\exp(|\boldsymbol{k}|^2/2). \tag{11}$$

s_k is related to the Fourier transform of the pair correlation function $g(r)$ (see e.g. [17]) which has in the thermodynamic limit the general analytic form [18]

$$g(r) = 1 - \exp(-r^2/2) + 2\exp(-r^2/4)\sum_m{}' (c_m/m!)(r^2/4)^m \tag{12}$$

where the prime at the sum indicates that it is restricted to odd m, $c_{2m} = 0$. Using this expression, one might calculate s_k and t_k to obtain finally

$$V_+(\boldsymbol{x}) = \int V(\boldsymbol{x}-\boldsymbol{x}')[a(|\boldsymbol{x}'|)/(1-f_0) + (1+f_0)\delta(\boldsymbol{x}')]\,\mathrm{d}^2x' \tag{13}$$

$$V_-(\boldsymbol{x}) = \int V(\boldsymbol{x}-\boldsymbol{x}')[a(|\boldsymbol{x}'|)/f_0 - f_0\delta(\boldsymbol{x}')]\,\mathrm{d}^2x' \tag{14}$$

where $a(r)$ is given by

$$a(r) = -(2\pi)^{-1}\sum_m{}' (c_m/m!)L_m(r^2)\exp(-r^2/2) \tag{15}$$

and $L_m(z)$ are Laguerre polynomials.

One obtains now an upper bound for the reduction of the energy gap

$$\Delta \leqslant (1/p)|\min_x V_+(x) + \min_x V_-(x)|. \tag{16}$$

This upper bound is a good estimate of the reduction of the energy gap in the case of substrate potentials that consist of long-range scatterers. If the range of the scatterers is long compared to the range of $a(r)$, one may replace $a(r)$ in the integrals in (13) and (14) by an appropriate multiple of $\delta(\boldsymbol{x})$. Using the first sum rule for the coefficient c_m given in [18]

$$\sum_m c_m = -f_0(1-f_0) \tag{17}$$

one obtains

$$V_+(\boldsymbol{x}) = V(\boldsymbol{x}) \qquad V_-(\boldsymbol{x}) = -V(\boldsymbol{x}). \tag{18}$$

Using these expressions, the estimate in (16) may be written as

$$\Delta \leqslant (1/p)|\max_x V(\boldsymbol{x}) - \min_x V(\boldsymbol{x})|. \tag{19}$$

Unfortunately it is not possible to give a good estimate of the right-hand side of (19) for the potentials used for the numerical calculations. The reason is simply that the best estimate of the maximum of a random potential is the maximum of all potentials of the ensemble. This must be calculated introducing correlations in the distribution of the positions of the scatterers as discussed above and it is therefore almost impossible to obtain the maximum of the ensemble. Consequently one has to use another class of bounded random potentials that do not contain macroscopic fluctuations. In the following we will use potentials that consist of spherically symmetric scatterers with the shape $v(r)$ on a lattice L with randomly distributed amplitudes

$$V(x) = \sum_{y \in L} V_y v(|\boldsymbol{x} - \boldsymbol{y}|). \tag{20}$$

$v(r)$ should satisfy the following conditions:

(i) $v(r)$ decays monotonically, $v(r) \geqslant v(r')$ if $r \leqslant r'$.
(ii) $v(0)$ is finite.
(iii) $v(r)$ is integrable in R^2.
(iv) $\Sigma_{y \in L}[v(|\boldsymbol{y}|) - v(|\boldsymbol{x} - \boldsymbol{y}|)] \geqslant 0$ for all x.

The last condition is used below to obtain an estimate of the maximum of $V(\boldsymbol{x})$. It is valid for a large class of shapes $v(r)$ if the range of $v(r)$ is smaller than or comparable to the lattice constants of L. For some $v(r)$ including Gaussian scatterers (iv) is valid independently of L.

The distribution of the amplitudes should be bounded. We choose the amplitudes to be distributed independently according to

$$P(V_y) = \tfrac{1}{2}[\delta(V_y - V) + \delta(V_y + V)]. \tag{21}$$

Γ may be estimated by the square root of the average of Γ^2

$$\Gamma^2 \approx \langle \Gamma^2 \rangle = V^2(|L|/A) \int [v(|\boldsymbol{x}|)]^2 \, \mathrm{d}^2 x. \tag{22}$$

Here $|L|$ denotes the number of scatterers and A denotes the area of the system. In the case of long-range scatterers, (22) may be written as

$$\Gamma^2 \approx V^2 \sum_{y \in L} [v(|y|)]^2. \tag{23}$$

Because of condition (iv) the right-hand side of (19) may be bounded from above using

$$[\max_x (V(x)) - \min_x (V(x))] \leqslant 2V \sum_{y \in L} v(y) \tag{24}$$

and one obtains

$$\Delta \approx (2/p)\Gamma \sum_{y \in L} v(|y|) \Big/ \Big(\sum_{y \in L} (v(|y|))^2 \Big)^{1/2}. \tag{25}$$

A similar result may be obtained for different bounded distributions of V_y. The essential point is that the substrate potential is bounded and long range and that the potential fluctuations do not become macroscopic. The main result contained in (25) is that the constant $C_{1/p}$ in (8) is proportional to $1/p$, i.e. to the fractional charge of the quasiparticles. Because of the positivity of $v(r)$ it is actually somewhat larger than $2/p$. This result agrees with the numerical results presented above and helps to understand them. In particular (25) shows that the reduction of the gap for $p = 5$ is a factor $3/5$ smaller than the reduction of the gap for $p = 3$. The numerical result for this ratio is roughly 0.7. The difference is easily explained by the fact that in the derivation of (25) we used the fact that the range of the scatterers is large compared to the range of $a(r)$. As the range of $g(r)$, the range of $a(r)$ is proportional to $p^{1/2}$. Thus the ratio of the ranges of $v(r)$ and $g(r)$ becomes smaller for larger values of p and the corrections to (25) are larger.

6. Comparison with experimental and other theoretical results

In the case of long-range scatterers, Γ is related to the single-particle relaxation time τ_{sp}

$$\Gamma \approx h/\tau_{sp}. \tag{26}$$

The strength of the disorder in a heterostructure is usually described by the mobility μ which is related to the mean scattering time τ_{sc} through

$$\tau_{sc} = m_{eff}\mu/e. \tag{27}$$

Unfortunately $\tau_{sp} = \tau_{sc}$ holds only in the case of δ-scatterers. Recently Gold [19] gave an extensive discussion of the various scattering mechanisms in two-dimensional electronic systems and their effect on this ratio. It turns out that the ratio τ_{sc}/τ_{sp} may be very large in the case of high-mobility GaAs–GaAlAs heterostructures. Therefore a direct quantitative comparison of our results with experiments is not possible. On the other hand (8) shows that for large magnetic fields, where the perturbative approach is valid, one obtains a constant reduction of the energy gap. This is consistent with the experimental results for the energy gap at different mobilities [3, 4].

Although the method to calculate the energy gap is different, we may compare our results with the results of Zhang *et al* [7] who calculated the reduction of the energy gap due to a single impurity. They diagonalized the Hamiltonian of four (toroidal geometry) to six electrons (spherical geometry) in the presence of a single Coulomb scatterer with a charge ze. For small z, the results are consistent with a linear behaviour of the reduction of the energy gap as a function of z. But the single impurity is localized in the plane where the electrons move, i.e. $d = 0$, and therefore a quantitative comparison is not

possible. The same problem occurs if one tries to compare our results with the results of Rezayi and Haldane [8]. Analytical results for the reduction of the energy gap due to disorder were obtained by MacDonald *et al* [9] and by Gold [10]. In both cases a single-mode approximation is used and the energy gap is not calculated as described above. On the other hand their results are consistent with a reduction of the energy gap proportional to Γ.

It should be mentioned that all the theoretical results concerning the reduction of the energy gap due to disorder are valid only for large enough magnetic fields. They should not be used to explain the collapse of the gap at lower magnetic fields (5 T for $f = 1/3$) since at low magnetic fields higher Landau levels become more important. Further at low magnetic fields (slightly above 5–6 T) the low lying excitations may be spin-reversed (see e.g. [1]) and are not well described by the magneto-roton theory used in [9, 10]. In the numerical calculations mentioned above [7, 8] the spin is not taken into account.

7. Conclusions

We have presented numerical as well as analytical results for the reduction of the energy gap in the fractional quantum Hall effect due to disorder. The results were obtained by first diagonalizing the Hamiltonian restricted to the lowest Landau level and by treating the disordered substrate potential as a small perturbation. If the substrate potential consists of long-range scatterers, the finite extension of the quasiparticle or the quasihole in space plays no significant role. They behave as electrons with a fractional charge. The reduction of the energy gap is determined by the mean value of the fluctuations of the potential alone as given in (8) and is proportional to the fractional charge of the quasiparticles.

Our result may be generalized to the case of spin-reversed excitations and to higher Landau levels. In these cases the shape of the function $a(r)$ in (15) is changed [20], but in the case of long-range scatterers the shape of $a(r)$ plays no significant role. This means that our results may be carried over to these cases. Nevertheless it should be noted that these cases are relevant for lower magnetic fields as pointed out above, so that higher orders of the perturbational series may become relevant. Higher orders may be treated using the effective potentials introduced in [20] and one should not expect a significant difference to our results. However, the collapse of the gap at low magnetic fields cannot be described within a perturbational treatment.

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